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RADIATIVE HEAT TRANSFER BETWEEN A FLUIDIZED BED AND A SURFACE

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Abstract—A model of radiative heat transfer in a dispersed medium is suggested which allows the calculation of the emissivity of an isothermal fluidized bed, the effective emissive ability of a non-isothermal bed and the temperature distribution near a heat transfer surface in a bed of a certain expansion when the radiative properties of the particles and the heat exchanger are prescribed.

NOMENCLATURE

d_p, particle diameter;
k, effective thermal conductivity of a gas interlayer between adjacent elementary layers of the model;
l, thickness of gas interlayer between adjacent elementary layers of the model;
m, porosity;
N, number of elementary layers in the model;
Q, radiation flux;
q, heat flux density;
q_p, radiosity of cell elements a, i, c, d induced by external radiation;
q'_p, radiosity of cell elements a', i', c', d' induced by external radiation;
q_{bs}, background radiosity of cell eleinents e, f, g, h;
q'_{bs}, background radiosity of cell elements e', f', g', h';
q_b, external radiation flux;
r, reflectivity;
r_i, reflectivity of i elementary layers of the model;
$r_i^{+(-)}$, reflectivity of i elementary layers of the model and of the bounding, $0(-)$ or $N+1(+)$, surface;
S_p, surface area of the cell elements a, i, c, d, a', i', c', d';
S_b, surface area of the cell elements e, f, g, h, e', f', g', h';
S_m, surface area of the cell element m;
T, temperature [K];
y_p, distance between centers of neighbouring particles in terms of their diameters;
t, temperature [°C].

Greek symbols

α, heat transfer coefficient;
δ_p, radiosity of cell elements a, i, c, d induced by background radiation;
δ'_p, radiosity of cell elements a', i', c', d' induced by background radiation;
ε, emissivity;
σ, Stefan–Boltzmann constant;
τ, transmissivity coefficient;
τ_i, transmissivity coefficient of i elementary layers of the model.

Subscripts

abs, absorbed;
b, black body;
e, effective;
f, incident;
fb, fluidized bed;
kk, convective–conductive;
p, particle;
r, radiative;
ref, reflected;
t, elementary layer;
trans, transmitted;
w, wall.

INTRODUCTION

THE RISE of temperature in fluidized-bed equipment has a double effect on the intensity of external heat transfer. Firstly, a change occurs in the thermophysical properties of a dispersed material and, particularly importantly, of a fluid. As a result, the hydrodynamics of the bed and its transport properties alter [1]. Secondly, the mechanism of energy transfer between a heat exchanger and a fluidized bed is made much more involved. The radiative transport, which is negligible in low-temperature systems, becomes significant. The radiative transport may comprise a number of phenomena [1], conditioned by the high concentration of scattering particles and the wave properties of radiation.

The effect of these phenomena is very difficult to take into account and at present there is virtually no relationship which would allow the calculation of the emissive ability of a fluidized bed and of the radiative fluxes over a wide range of particle properties. (The estimates reported [1, 2] have been made on the assumption that the particles forming the bed are ideally black.)

At the same time, a technique for computing radiation transfer in a fluidized bed in a wide range of particle properties would be of great value, particularly in view of the fact that particles of metal oxides, commonly used as high-temperature heat-transfer agents, are characterized by a high reflectivity, and the black body approximation is inapplicable.

The objectives of this study were to determine more precisely the basic features of radiation transfer in a fluidized bed, to choose an adequate model and, based on it, to derive the relationships that are required to calculate the radiative component of the external heat transfer coefficient.

SPECIFIC FEATURES OF RADIATION TRANSFER IN A FLUIDIZED BED

Most of the phenomena that may occur in radiation propagation in a fluidized bed owe their origin to the wave nature of radiation [1]. Their quantitative description is most elaborate, but they can be shown to be of no importance in such a medium as a fluidized bed.

In order that the radiation interaction with a separate particle, which is determined by the relationship between the wave length λ and the size of particles as well as their concentration, can be estimated on the basis of the specific features of a high-temperature fluidized bed [1, 3, 4] the following ranges for the quantities λ, d_p, y_p have been chosen:

$$\lambda < 10\,\mu\text{m},\quad d_p > 50\,\mu\text{m},\quad y_p = 1\text{–}2. \tag{1}$$

These inequalities show that:

(1) The scattering parameter $\pi d_p/\lambda > 15$ and the particles can be considered as large enough from the view-point of the scattering theory [5].

(2) Perturbations of the wave front on the particles have a diffraction nature and may be neglected because of the small characteristic distances y_p. (This follows from the estimates made by the formulae that were suggested in ref. [6].)

(3) Solid materials used in fluidization may be considered as gray within the range of λ given in (1).

An irregularly and continually changing arrangement of particles in a fluidized bed, their optically roughness surface (as a result of crushing) and large dimensions allow the conclusion that the cooperative interference effects in the system considered are unimportant [7].

Thus, the radiative transfer in a fluidized bed may be regarded as occurring due to multiple reflection by particles (which may also transmit a portion of the incident radiation). Moreover, the estimates show that for a quantitative description of this process the concepts of geometrical optics can be used.

Certain information on the specific features of radiative transfer in a fluidized bed exists in the published literature on the experimental investigation of this process. It follows from these works that:

(1) The radiative flux is independent of the dimensions of particles [8, 9], which leads to the importance of radiative heat transfer in large particle beds.

(2) The radiative flux is independent of the velocity of a fluid (at the fluidization number $\gtrsim 3$) [10] and the heat transfer surface location in a bed [11] and, consequently, of the bed structure near the surface which is determined by these parameters.

(3) The radiative flux is independent of the radiative properties of a fluid [11].

The experimental results show that the basic characteristics of a fluidized bed that determine its radiative properties are the emissivity of the particles forming the bed and temperature distribution in the system [11].

This analysis of the reported experimental data and of the specific features of a fluidized bed allows us to assume that the calculation of the emissivity of a bed and radiative transfer in it can be based on a representation of the bed as an assembly of large spherical particles with diffuse-gray surface in a transparent medium.

THE CALCULATION TECHNIQUE OF RADIATIVE TRANSFER IN A FLUIDIZED BED

Calculations of radiation transfer in both homogeneous and rarefied dispersed media are usually made by using the radiative transfer equation. Then the medium is characterized by the absorption and scattering coefficients, the emissive ability and the phase function [5, 12]. However, in dense dispersed media, such as a fluidized bed, {with the porosity range from 0.4 to 0.93 [4] and the distance between particles as determined by equation (1)}, the radiative transfer equation is of limited usefulness, since the conditions for its applicability are not fulfilled [13, 14].

For this reason it seems more suitable to use another familiar method of calculation, based on the development of a special model of the medium. Of the models used for the description of radiation transfer in packings [15–17, 26] the most appropriate for a fluidized bed seems to be that used in optics, i.e. the pile model [16, 17]. There, a dispersed medium is represented by an assembly of plane-parallel reflecting, absorbing and radiating plates (Fig. 1). The plates that form the pile, i.e. the elementary layers, are characterized by their reflection and absorption coefficients and by their emissivities, which depend on the properties and concentration of particles forming the dispersed medium. In a familiar version of this model for the case of a packing, it was assumed that each elementary layer has the thickness d_p and the optical characteristics of the particle material. The concentration of

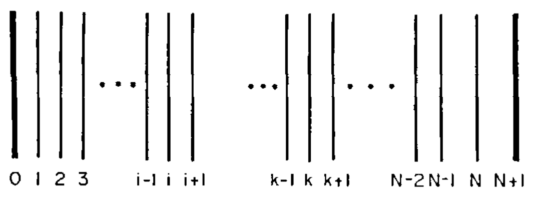

FIG. 1. Model of radiation transfer in a fluidized bed. The pile model.

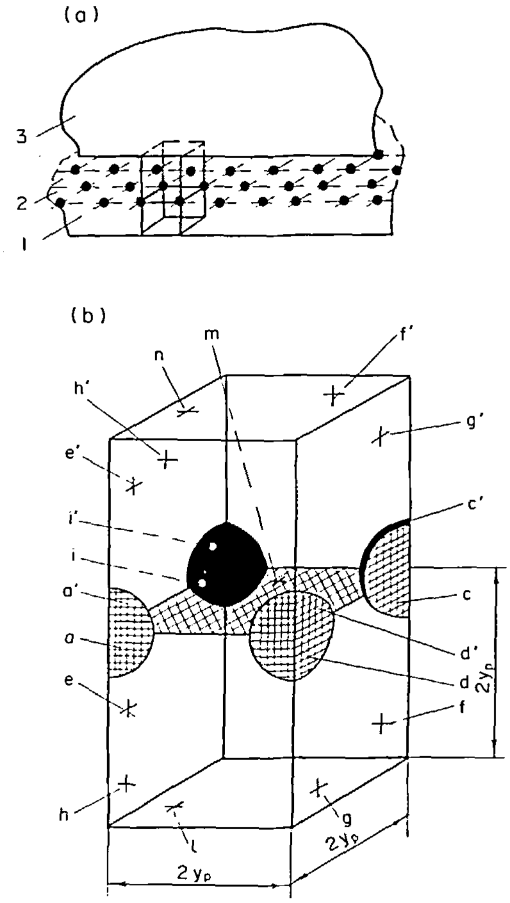

FIG. 2. Model of an elementary layer of the pile: (a) a system to determine the optical characteristics of a 2-dim. dispersed medium; (b) a cell of the system (a).

particles in a fluidized bed varies within a wide range and this kind of relationship between the characteristics of the elementary layer and particles is inapplicable. Consequently, the pile model can be applied to a fluidized bed if the elementary layer characteristics are determined as functions of the properties of particles for a wide range of particle concentrations.

Table 1. Designations of the view factors for the elements of the cell φ_{i-j}

i	j									
	a	i	c	d	e	f	g	h		m
a		P	Q	P	H	G	G	H	T	C
i	P		P	Q	H	H	G	G	T	C
c	Q	P		P	G	H	H	G	T	C
d	P	Q	P		G	G	H	H	T	C
e						Y	Z	Y	K	
f					Y		Y	Z	K	
g					Z	Y		Y	K	
h					Y	Z	Y		K	
m	C_r	C_r	C_r	C_r	L	L	L	L	M	

RADIATIVE PROPERTIES OF A TWO-DIMENSIONAL DISPERSED MEDIUM

An elementary layer of the pile is a flat uniform plate which is equivalent to a layer of dispersed material formed by the particles located in about the same plane drawn parallel to the surface of the dispersed medium. The elementary layer thus defined is a model of a 2-dim. dispersed medium. In order to determine its optical characteristics in a wide range of properties of particles and their concentration, the system is used as shown in Fig. 2(a). It is made up of two ideally black planes, 1 and 3, between which a model 2-dim. dispersed medium is located, i.e. a system of spheres arranged regularly at the nodes of a plane square-mesh grid.

For the calculation of the optical characteristics of a 2-dim. dispersed medium, i.e. the reflection, transmission and absorption factors, it was assumed that on plane 1 (Fig. 2a) a radiative flux with the surface density q_b is prescribed, which is external relative to model 2, with self-emission of the latter being assumed to be zero. As a consequence of the scattering of q_b by the particles of model 2, a certain portion of it returns to surface 1 in the form of the flux reflected by the model, a portion is transmitted to plane 3, and the remaining radiation is absorbed by the particles of model 2. Thus, the reflectivity and transmissivity factors, the absorption coefficient and the emissivity (the latter two being of the same magnitude in accordance with the Kirchhoff law [5]) of the 2-dim. dispersed medium model can be defined as the ratios of respective fluxes in the system of Fig 2(a) to the external flux of density q_b incident on it from the side of plane 1.

As has been shown above, in order to calculate the optical characteristics of the system at hand we may confine ourselves to the concepts of geometrical optics. This allows an assumption that the spheres forming the model have unit radius, while the distance between the planes in the system of Fig. 2(a), just as the grid spacing in model 2, is equal to $2y_p$. (The system then becomes more symmetric.)

A regular arrangement of spheres in the model allows a further simplification of the calculation. A transition is made from the infinite system of Fig. 2(a) to one of its cells represented in Fig. 2(b). This cell is made up of the portions of black planes 1 and 3 (faces l and n), 1/8 fractions of spheres (a, i, c, d, a', i', c', d') and auxiliary black faces e, f, g, h, e', f', g', h'. The cell will now be considered as a set of two closed systems comprised of black and gray diffuse isothermal surfaces (a, i, c, d, e, f, g, h, l, m and a', i', c', d', e', f', g', h', n, m, respectively).

The mathematical apparatus employed to calculate the radiation transport in these systems is described in ref. [5]. The incorporated view factors for all the pairs of surfaces forming the upper and lower half of the cell have been calculated to within 1%. Table 1 contains abbreviated designations for these factors, while Figs.

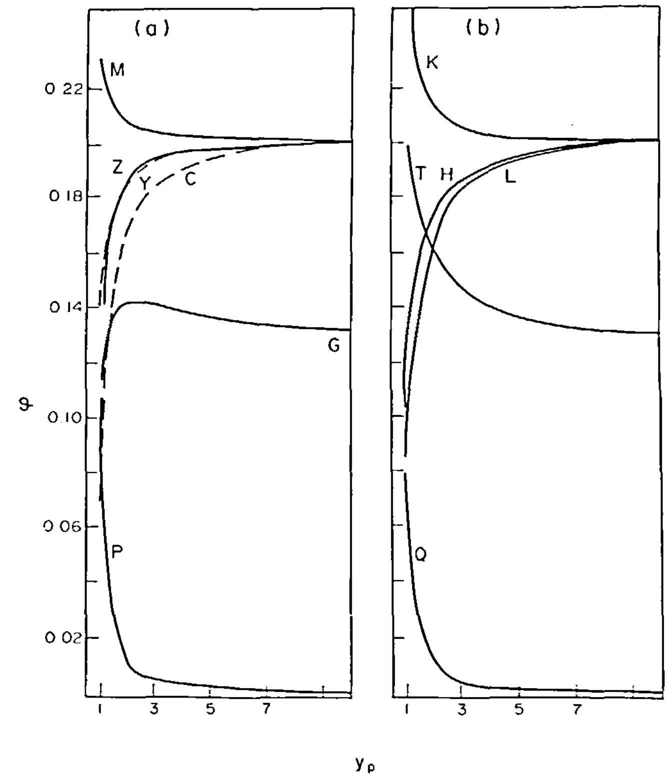

FIG. 3. Dependence of the view factors of cell elements on the cell dimensions.

3(a) and (b) show the dependence of their magnitudes on the cell dimensions (parameter y_p). In transition from an infinite system to a cell (Fig. 2b) it turns out to be necessary to prescribe the external radiation with density q_b not only at the face l of plane 1, but also on the side faces e, f, g, h in order to account for the contribution of the portion of plane 1 located beyond the cell. Moreover, it is essential that at the side faces e, f, g, h and e′, f′, g′, h′, the background radiation with densities q_{bs} and q'_{bs}, respectively, be prescribed in order to account for the effect of particles comprising the model but not entering into the cell. Due to the additivity of the thermal radiation fluxes, the propagation of the external and background radiation in the cell can be considered separately.

When the external radiation is being scattered in the cell, the flux density on the black surfaces e, f, g, h, l and e′, f′, g′, h′, n is set (q_b and 0, respectively), while that on the surface m can be eliminated from consideration as an intermediate quantity. The gray surfaces a′, i′, c′, d′ of the upper portion of the cell (a, i, c, d in the lower portion) are identical, and determination of the flux density for one of these is sufficient. Thus, the external radiation transport is described by a system of two equations, which express the radiosity of the gray surfaces of the upper and lower parts of the cell, respectively, as a sum of the reflected and transmitted (by this surface) radiation from all the remaining elements of the cell

$$a_1 q'_p - a_3 q_p = a_5 q_b,$$
$$- a_3 q'_p + a_1 q_p = a_4 q_b. \tag{2}$$

where

$a_1 = 1 - r_p(2P + Q),\ a_2 = T + 2(G + H),$

$a_3 = 3CC_r r_p,\ a_4 = a_2 \tau_p,\ a_5 = a_2 \tau_p + 2LCr_p,$

in which the subscript r means that the reciprocal view factor has been used.

The solution of equation (2) is

$$q'_p = \frac{a_1 a_5 + a_3 a_4}{a_1^2 - a_3^2},\ q_p = \frac{a_1 a_4 + a_3 a_5}{a_1^2 - a_3^2}. \tag{3}$$

The background radiosity of the identical black faces of the upper (e′, f′, g′, h′) and lower (e, f, g, h) portions of the cell is unknown. For this to be determined, the condition for the net background radiation flux through any end face to be equal to zero due to the translative symmetry of a 2-dim. dispersed medium can be used. On the black surfaces l and n the density of the background radiation is equal to zero. The background radiation-induced radiosity of the grey surface of particles can be written in the same form as in the case of external radiation scattering equation (2). Thus, the background radiation transport can be calculated from the following system of four equations:

$$
\begin{aligned}
&q_{bs} - 3a_7 L q'_{bs} - a_6\delta_p - 2a_7 C_r \delta'_p = a_6 q_p + 2a_7 C_r q'_p, \\
&3a_7 L q_{bs} + q'_{bs} - 2a_7 C_r \delta_p - a_6 \delta'_p = 2a_7 C_r q_p + a_6 q'_p, \\
&-a_9 q_{bs} - a_8 q'_{bs} + a_1 \delta_p - a_{10}\delta'_p = 0, \\
&-a_8 q_{bs} - a_9 q'_{bs} - a_{10}\delta_p + a_1 \delta'_p = 0 \qquad (4)
\end{aligned}
$$

here

$$
a_6 = 2\frac{S_p(G+H)}{S_b(1-2Y-Z)}, \qquad a_7 = \frac{S_m L}{S_b(1-2Y-Z)},
$$

$$
a_8 = 2\tau_p(G+H) + 2r_p LC,
$$

$$
a_9 = 2\tau_p LC + 2r_p(G+H),
$$

$$
a_{10} = \tau_p(2P+Q) + 3r_p CC_r.
$$

Knowing the solution of the system of equations 2)–(4), one can calculate the flux reflected by model 2 ncident on the face 1), the flux transmitted to the face n f the cell and that absorbed by model 2 from the ollowing formulae

$$
\begin{aligned}
Q_{ref} = {} & 4\{S_p[T(q_{pr} + \delta_p) + q_{p\tau}] + S_b K q_{bs} + S_m \\
& \times [(M + 2L + C_r) C_r (q'_{pr} + \delta'_p) \\
& + (L + 2C_r + M) L q'_{bs}]\}, \\
Q_{trans} = {} & S_m[4L(4L + M) + 8C_r L + M] q_b + 4 \\
& \times \{S_p[T(q'_{pr} + \delta'_p) + q'_{p\tau}] + S_b K q'_{bs} + S_m \\
& \times [(M + 2L + C_r)(q_{pr} + \delta_p) \\
& + (L + 2C_r + M) L q_{bs}]\}, \\
Q_{abs} = {} & 4\varepsilon_p S_p[(a_2 + CL) q_b + 2(G + H + CL) \\
& \times (q_{bs} + q'_{bs}) + (2P + Q + 3CC_r) \\
& (q_{pr} + q'_{pr} + \delta_p + \delta'_p)],
\end{aligned}
$$

where

$$
q_{pr} = \frac{a_1 a_2 + (2L + 3C_r)C}{a_1^2 - a_3^2} r_p q_b,
$$

$$
q_{p\tau} = \frac{a_2 a_4 \tau_p}{a_1^2 - a_3^2} q_b,
$$

$$
q'_{pr} = \frac{2LCa_1 + a_2 a_3}{a_1^2 - a_3^2} r_p q_b,
$$

$$
q'_{p\tau} = \frac{a_1 a_2 \tau_p}{a_1^2 - a_3^2} q_b,
$$

$$
q_{pr} + q_{p\tau} = q_p, \qquad q'_{pr} + q'_{p\tau} = q'_p.
$$

Thereafter the optical characteristics of the 2-dim. dispersed medium and, consequently, of the elementary layer of the pile are calculated from

$$
r_t = \frac{Q_{ref}}{Q_f}, \quad \tau_t = \frac{Q_{trans}}{Q_f}, \quad \varepsilon_t = \frac{Q_{abs}}{Q_f}. \qquad (6)
$$

where

$$
Q_f = [4S_p a_2 + (4L + M) S_m] q_b
$$

is the radiation flux from the black plane 1 incident on the portion of model 2, which enters into the cell (Fig. 2a).

The results of the calculation, i.e. the reflectivity and transmissivity factors and the emissivity of the elementary pile layer, are presented in Fig. 4. As is seen from this figure, the optical coefficients of the elementary layer depend strongly on the distance between the particles within the range of densities which is typical of a fluidized bed and, to a great extent, are determined by the properties of the particles forming the bed. As the distance between the particles, y_p, increases, the transmissivity and reflectivity factors and the emissivity of the elementary layer tend to their obvious limits (r_t, $\varepsilon_t \to 0$, $\tau_t \to 1$).

The calculated characteristics of the elementary layer of the pile in a wide range of the properties of particles and their concentration (the calculations of the 2-dim. dispersed medium characteristics are also given in the works of the present authors [18, 19]) allow us to turn to the study of radiation transport in a fluidized bed using the pile model.

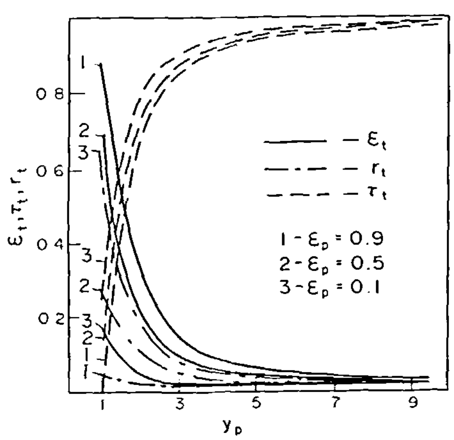

FIG. 4. Dependence of the optical characteristics of a 2-dim. dispersed medium (of an elementary layer) on the distance between particles.

RADIATIVE HEAT TRANSFER BETWEEN AN ISOTHERMAL FLUIDIZED BED AND A DISTANT SURFACE

When the surface, which takes part in radiative exchange with the bed, is removed to some distance away from it, the fluidized bed may be regarded as isothermal due to the agitation of particles. A temperature gradient within the dispersed medium is absent then, and radiative heat transfer in the system can be considered as occurring between two surfaces, each of which has its own emissivity and temperature. The density of the net flux can be calculated in this case by the following well-known formula [5]:

$$q_r = \sigma \varepsilon_{net}(T_{fb}^4 - T_w^4),$$

where

$$\varepsilon_{net} = \left(\frac{1}{\varepsilon_{fb}} + \frac{1}{\varepsilon_w} - 1\right)^{-1}.$$

Here, the unknown quantity is the emissivity of the fluidized bed surface. For this to be found, we may use the pile model, the elementary layers of which are characterized by the optical characteristics determined above. Since the fluidized beds are usually very thick compared with the size of particles, the following limiting relations, which determine ε_{fb}, hold:

$$\lim_{n\to\infty} r_n = r_{fb}, \quad \lim_{n\to\infty} \tau_n = 0, \quad r_{fb} + \varepsilon_{fb} = 1 \quad (8)$$

where [26]

$$r_n = r_{n-1} + \frac{\tau_{n-1}^2 r_t}{1 - r_{n-1} r_t},$$

$$\tau_n = \frac{\tau_{n-1}\tau_t}{1 - r_{n-1} r_t}.$$

The optical characteristics of the elementary pile layer and of the entire dispersed medium model are determined not only by the particle material properties but also by their concentration. At the prescribed mean porosity of the fluidized bed a dimensionless distance between the centers of the neighbouring particles can be calculated by the following formula which was derived from the expression of porosity for a cubic array of particles [1, 3, 4] ($m \geq 0.48$):

$$y_p = [\pi/6(1 - m)]^{1/3}. \quad (9)$$

The fluidized bed surface emissivity calculated by equations (8) as a function of the emissivity of particles and the distance between them is presented in Figs. 5 and 6. As is seen from these figures, the emissivity ε_{fb} is mainly determined by the material properties of particles and, to a much lesser degree, by their concentration, which agrees with the results of experimental investigations. When the distance between neighbouring particles $y_p \geq 2$, its further increase will actually have no effect on ε_{fb}.

As is seen from Fig. 6, the results of experimental studies from refs. [25, 27–30] (the emissivities of particles are borrowed from refs. [1, 28, 31]) are well described by curve 1 for an expanded bed (at $y_p \geq 2$), with the mean deviation not exceeding 10%.

Estimates made by the two-phase theory have shown that when a bed exchanges radiation with a surface of small dimensions, that are commensurate with the mean size of bubbles (e.g. radiometer peep-hole), then the mean porosity near it is rather high. Presumably, the emissivity of the fluidized bed surface may be regarded in this case as equal to its limiting value for an expanded system.

Thus, one may conclude that the model suggested allows the accurate calculation of the emissive ability of a fluidized bed, and the correlations obtained can be used to calculate the value of ε_{fb} provided the emissivity of the particles forming the bed is known.

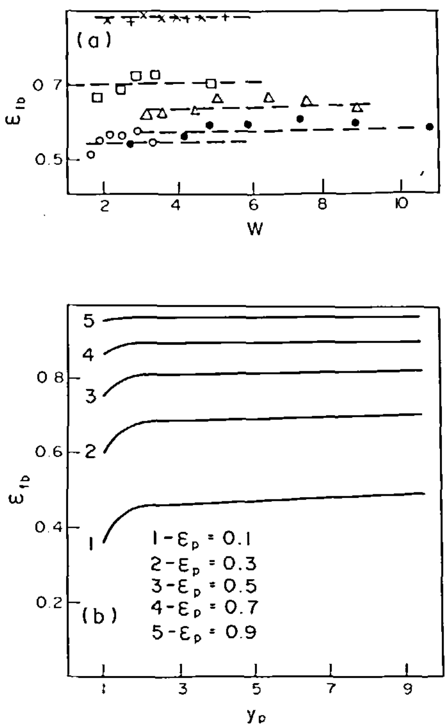

FIG. 5. Experimental (a) [25] and predicted (b) dependences of the isothermal fluidized bed surface emissivity on bed expansion.

RADIATIVE HEAT TRANSFER BETWEEN A NON-ISOTHERMAL FLUIDIZED BED AND A SUBMERGED SURFACE

Let a heat transfer surface be immersed in a fluidized bed and the temperature of it be different from that of the bed core. As a result of the agitation of particles and energy exchange between the particles, the gas and heat exchanger, a certain temperature profile is developed near the heat exchanger surface. The temperature gradient near this surface is smaller in this case than in an isothermal bed, and the possible heat fluxes decrease. Special investigations [20] have shown, in particular, that the radiative flux emitted by the bed depends markedly on the temperature distribution near the heat transfer surface. The non-isothermicity of the bed causes a change in the magnitude of the radiative flux. To account for this effect, it has been suggested [20] that in equation (7) the effective emissivity of the fluidized bed surface should be used instead of the true value of ε_{fb}. However, a rigorous derivation of the formula for q_r, which takes into account the fact that for the effective emissive ability the relation $1 - r_{fb} = \varepsilon_{fb} \neq \varepsilon_e$ is valid, leads to the following expression, which differs from equation (7):

$$q_r = \sigma\, \varepsilon_{net} \left(\frac{\varepsilon_e}{\varepsilon_{fb}} T_{fb}^4 - T_w^4 \right) \qquad (10)$$

where ε_{net} is calculated in the same way as for equation (7).

The suggested effective emissivity of the bed is a function of a number of the system parameters: $\varepsilon_e = \varepsilon_e (T_w, T_{fb}, \varepsilon_w, \varepsilon_p, m)$. Certain information on ε_e is given in ref. [20] in the form of the experimental function $\varepsilon_e (T_w)$ at several values of T_{fb}. However, these data are too few to reveal the actual form of the function ε_e and its coupling with such system parameters as ε_w, ε_p, m. It turns out to be possible to determine the value of ε_e as a function of all the basic parameters from the solution of the problem on radiative transport in a dispersed medium under steady-state conditions with the aid of the pile model. In this case the non-isothermal zone between the heat transfer surface and the bed core is represented by an assembly of N elementary layers (Fig. 1), the optical characteristics of which are determined, as shown above, by the properties of the particles and their concentration.

The heat transfer surface is represented in the model by the 0th plane with the reflectivity r_w and temperature T_w; the core is represented by the $(N + 1)$th plane having the parameters r_{fb}, T_{fb} ($\tau_{fb} = 0$).

In the process of external heat transfer of a fluidized bed a substantial part is played by a convective–conductive transport. The latter should therefore be considered together with the radiation transfer. In the pile model, the convective–conductive component is accounted for as thermal conductivity of gaseous interlayers between the elementary layers, the thickness of which is determined as [21]

$$l = (y_p - 0.87) d_p. \qquad (11)$$

Since the thermal resistance of the fluidized bed is offered mainly by the gas, the thermal conductivity of the elementary layers is assumed to be infinitely high and the effective thermal conductivity of the gas interlayer between them is determined by the external heat transfer coefficient calculated from any of the relations of refs. [3, 20, 22] as

$$k = \alpha_{kk}\, l (N + 0.5). \qquad (12)$$

It is assumed in this formula that the gas layer nearest to the heat exchanger is half as thick as all the remainder layers. This determination of the effective thermal conductivity makes it possible to calculate a change in α_{kk} caused by radiative transfer.

On the average, the temperature distribution in a non-isothermal zone of the bed is steady, and in order to derive the governing system of equations one may use the energy balance relations. In this case the energy radiated by the ith elementary layer in both directions ($2\sigma\, \varepsilon T_i^4$) is equated to the sum of the absorbed fraction of radiative fluxes from all the remaining elementary layers and the difference between the conductive fluxes on the left and on the right from the selected elementary layer

$$\sum_{k=1}^{N} [\delta_{ik}(2 - r_{k-1}^- a_k^- - r_{N-k}^+ a_k^+) - E(i-k) \times (\beta_{ik}^- + \beta_{ik}^+) a_k^+ - E(k-i)(\gamma_{ik}^- + \gamma_{ik}^+) a_k^-] x_k^4 + \mu_i x_{i+1} + \nu_i x_{i-1} - (\mu_i + \nu_i) x_i = (\gamma_{iN+1}^- + \gamma_{iN+1}^+) \times \varepsilon_{fb} + (\beta_{i0}^- + \beta_{i0}^+) \varepsilon_w x_0^4, \; i = 1, \dots, N \qquad (13)$$

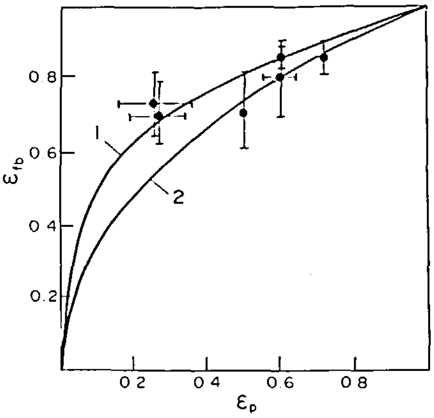

FIG. 6. Dependence of the expanded (1) and packed (2) bed emissivity on the emissivity of particles.

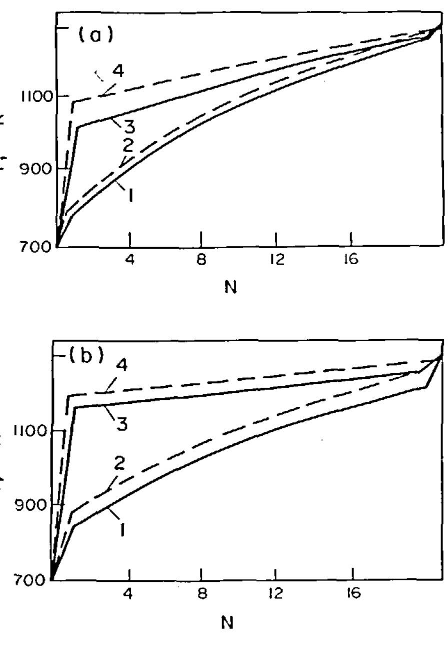

FIG. 7. Temperature distribution in a non-isothermal zone of the fluidized bed in the case of radiative transfer: (a) $y_p = 1$; (b) 2; 1, $r_w = 0.1$; $\varepsilon_p = 0.1$; 2, $r_w = 0.1$; $\varepsilon_p = 0.9$; 3, $r_w = 0.9$; $\varepsilon_p = 0.1$; 4, $r_w = 0.9$; $\varepsilon_p = 0.9$.

where

$$x_i = T_i/T_{fb},\ x_0 = T_w/T_{fb},\ \mu_i = k/\sigma\, l_{i,i+1}\, T_{fb}^3,$$

$$\nu_i = k/\sigma\, l_{i,i-1}\, T_{fb}^3,\ a_k^+ = \varepsilon_k\left[\frac{1}{1 - r_k^- r_{N-k}^+} + \frac{\tau}{(1 - r_{N-k}^+ r)(1 - r_{k-1}^- r_{N-k+1}^+)}\right]$$

$$a_k^- = \varepsilon_k\left[\frac{1}{1 - r_{k-1}^- r_{N-k+1}^+} + \frac{\tau}{(1 - r_{k-1}^- r)(1 - r_k^- r_{N-k}^+)}\right],$$

$$\beta_{ik}^+ = r_{N-i}\frac{\tau_{N-k}(1 - r_{N-i} r_{fb})}{\tau_{N-i}(1 - r_{N-k} r_{fb})},$$

$$\beta_{ik}^- = \frac{\tau_{N-k}(1 - r_{N-i+1} r_{fb})}{\tau_{N-i+1}(1 - r_{N-k} r_{fb})},$$

$$\gamma_{ik}^+ = \frac{\tau_{k-1}(1 - r_i r_w)}{\tau_i(1 - r_{k-1} r_w)},$$

$$\gamma_{ik}^- = r_{i-1}^-\frac{\tau_{k-1}(1 - r_{i-1} r_w)}{\tau_{i-1}(1 - r_{k-1} r_w)},$$

$$\delta_{ik} = \begin{cases}0, & i \neq k\\ 1, & i = k\end{cases},\quad E(i) = \begin{cases}0, & i \leqslant 0\\ 1, & i > 0\end{cases}.$$

The nonlinear system (13) can be solved by the Newton method which is quite stable for this case. The solution of equation (13) at the prescribed porosity, the properties of particles and of the surface is the temperature distribution within the non-isothermal zone of the prescribed depth N. Some of the possible temperature distributions are presented in Figs. 7 and 8. The temperature profiles (Fig. 8) vary markedly during the transition from packed to rarefied systems. This occurs because of a change in the relative contribution of the radiative and convective-conductive transfer. With an increase in the porosity of the bed, the major contribution comes from radiation, since in this case the number of layers participating in radiative exchange increases, and the distribution of temperature at a marked rarefaction approaches that in the case of purely radiative transfer (Fig. 7). In transition to weakly concentrated systems, the radiative properties of particles and of heat transfer surface become substantial.

The calculated temperature distribution near the surface allows determination of the heat fluxes. The convective-conductive flux is calculated by the formula

$$q_{kk} = \frac{2k}{l}(T_1 - T_w). \qquad (14)$$

The radiative flux emitted by any of the model planes to the side of the 0th $(-)$ and $N + 1$th $(+)$ surfaces is determined by the following expression:

$$q_i^\pm = \sigma\left[T_w^\pm \sum_{k=0}^{N+1} c_{ik}^\pm + (T_{fb}^\pm - T_w^\pm)\sum_{k=0}^{N+1} c_{ik}^\pm \theta_k\right]. \qquad (15)$$

where

$$c_{ik}^+ = \delta_{ik} a_k^+ + E(i - k)\frac{\beta_{ik}^+}{r_{N-i}^+} a_k^+ + E(k - i)\gamma_{ik}^+ r_i^- a_k^-,$$

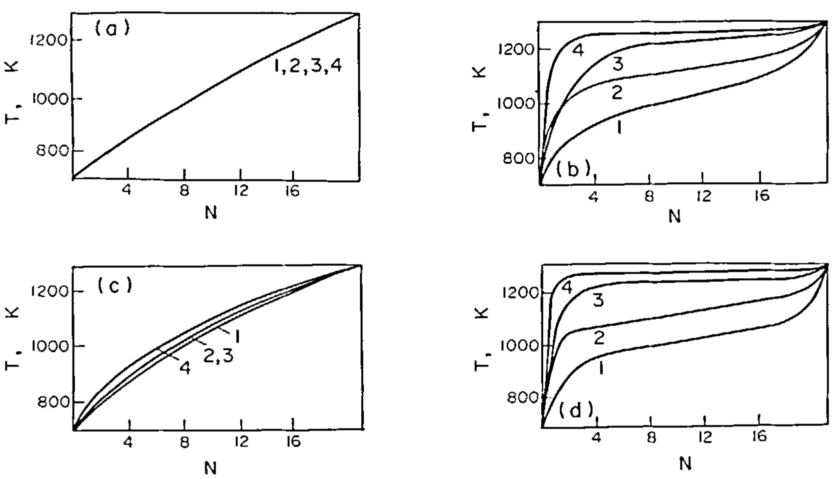

FIG. 8. Temperature distribution in a non-isothermal zone of the fluidized bed in the case of radiative–conductive transfer: (a, b) $d_p = 0.5$ mm; (c, d) $d_p = 2$ mm; (a, c) $y_p = 1.01$; (b, d) $y_p = 5$. 1, 2, $r_w = 0.1$; 3, 4, 0.9; 1, 3, $\varepsilon_p = 0.1$; 2, 4, $\varepsilon_p = 0.9$.

$$c_{ik}^- = \delta_{ik} a_k^- + E(i-k)\beta_{ik}^- r_{N-i+1}^+ a_k^+ + E(k-i)\frac{\gamma_{ik}^-}{r_{i-1}^-} a_k^- ,$$

$$\theta_k = (T_k^4 - T_w^4)/(T_{fb}^4 - T_w^4),$$

and is a function of the radiative properties and temperature of all the model elements.

The radiative flux emitted by the fluidized bed to the side of the submerged surface is equal to the value of q_1^- calculated from equation (15). The ratio of q_1^- to the flux from the isothermal layer will yield the following expression for the effective emissive ability of the non-isothermal fluidized bed:

$$\frac{\varepsilon_e}{\varepsilon_{fb}} = (1 - A)(T_w/T_{fb})^4 + A, \tag{16}$$

where

$$A = \sum_{k=1}^{N+1} c_{1k}^- \theta_k .$$

The coefficient A in equation (16) in the case of a purely radiative exchange is independent of the temperatures T_w and T_{fb}, and the quantity ε_e is a linear function in the coordinates $\varepsilon_e/\varepsilon_{fb}$ and $(T_w/T_{fb})^4$ [23, 24]. Moreover

$$A = A(\varepsilon_w, \varepsilon_p, m, N). \tag{17}$$

In the case of a radiative-conductive exchange, the dependence (16) becomes more complex in virtue of the nonlinear nature of the system (13) and the coefficient A turns out to be a function of a large number of parameters

$$A = A(\varepsilon_w, \varepsilon_p, m, N, T_w, T_{fb}). \tag{18}$$

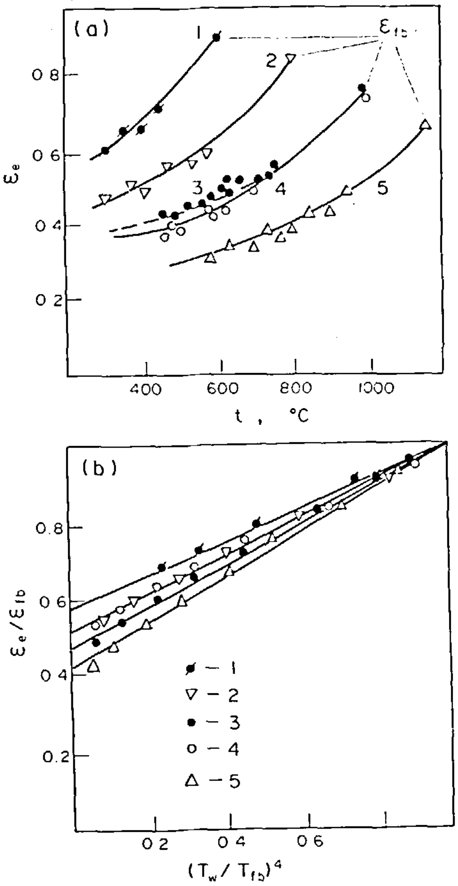

FIG. 10. An experimental dependence of the effective emissivity of a non-isothermal fluidized bed on the immersed surface temperature [20]: (a) in the coordinates of ref. [20]; (b) in the coordinates of equation (16); 1, t_{fb} = 600°C; 2, 800; 3, 4, 1000°C, 5, 1225°C; 1, 2, 4, d_p = 0.32 mm; 3, 5, 0.5 mm.

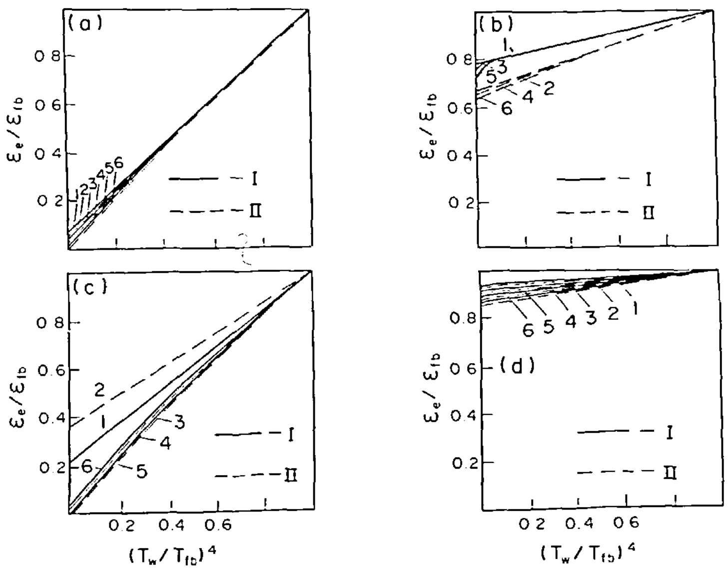

FIG. 9. The effective emissivity of a non-isothermal fluidized bed vs the immersed surface temperature: N = 20; (a, b) r_w = 0.1; (c, d) r_w = 0.9; (a, c) y_p = 1; (b, d) y_p = 5; I, ε_p = 0.1; II, ε_p = 0.9; 1, 2, radiation; 3, 4 (d_p = 0.5 mm) and 5, 6 (d_p = 1.0 mm), radiation and heat conduction.

Figure 9 shows the results of calculation of ε_e by equation (16) for the cases of radiative and complex exchange at different values of the system parameters. As is seen from this figure, the convective–conductive transfer in a packed system almost completely suppresses the effect of radiative properties of the particles and of the wall. In a rarefied system, the effect of ε_p, ε_w on the function ε_e is much more noticeable, while the nonlinear nature of the system (13) is manifested only at sufficiently large difference of temperatures T_w, T_{fb} ($T_w/T_{fb} < 0.6$).

Figure 10 shows the results of an experimental investigation of the function ε_e (T_w, T_{fb}), given in ref. [20], in the initial coordinates and, according to equation (16), in the coordinates $\varepsilon_e/\varepsilon_{fb}$ and $(T_w/T_{fb})^4$. Comparison of Figs. 9 and 10 shows that the equation obtained by the suggested model allows a rather accurate calculation of the effective emissive ability of a nonisothermal fluidized bed. Then, taking into account that the non-linearity is manifested only at a sufficiently large difference between the temperatures T_w and T_{fb}, the calculation of ε_e can be carried out in a more simple linear radiative approximation (at $T_w/T_{fb} > 0.5$).

As is seen from equation (16), the effective emissivity is an integral characteristic of the temperature distribution near the heat exchanger surface. A rather accurate, as follows from Figs. 9 and 10, description of the experimental function ε_e (T_w, T_{fb}) allows the use of the equations obtained, together with the experiment similar to that described in ref. [20], to calculate the temperature profile and the depth of the non-isothermal zone. Thus, the estimates made for the condition of the experiment described in ref. [20] have shown that the thickness of the non-isothermal zone is equal to about 10–15 particle layers with appreciable cooling of particles from 150 to 400 K for the first (from the wall) layer of particles at $T_w = 573$ K and $T_{fb} = 873$–1498 K.

CONCLUSION

A qualitative and quantitative confirmation of the predictions by using the available experimental data shows that the model suggested gives a fairly true picture of radiation transfer in a fluidized bed. Its development was favoured by a rigorous determination of the ranges of basic process parameters, elucidation of the pertinent properties of the dispersed medium from the available experimental data, and assessment of the applicability of different methods used for radiation transport calculation. A relative simplicity of the model has made it possible to obtain the results (emissivity, effective emissive ability) that can be directly used in the engineering practice which employs the well-known relations for the calculation of radiative fluxes.

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TRANSFERT RADIATIF ENTRE UN LIT FLUIDISE ET UNE SURFACE

Résumé—Un modèle de transfert radiatif dans un milieu dispersé est suggéré pour calculer l'émissivité d'un lit fluidisé isotherme, l'aptitude à émettre d'un lit non-isotherme et la distribution de température près d'une surface dans le lit quand les propriétés radiatives des particules et de l'échangeur de chaleur sont données.

STRAHLUNGSAUSTAUSCH ZWISCHEN EINEM FLIESSBETT UND EINER OBERFLÄCHE

Zusammenfassung—Es wird ein Modell des Strahlungswärmeaustausches in einem dispersen Medium vorgeschlagen, mit dessen Hilfe es möglich ist, das Emissionsvermögen eines isothermen bzw. das effektive Emissionsvermögen eines nicht-isothermen Fließbettes und die Temperaturverteilung an einer wärmeübertragenden Fläche bestimmter Ausdehnung zu berechnen, wenn die Strahlungseigenschaften der Partikel und des Wärmeaustauschers gegeben sind.

ЛУЧИСТЫЙ ТЕПЛООБМЕН ПСЕВДООЖИЖЕННОГО СЛОЯ С ПОВЕРХНОСТЬЮ

Аннотация—В работе предлагается модель переноса излучения в дисперсной среде, позволяющая вычислять радиационные характеристики псевдоожиженного слоя: степень черноты изотермичного псевдоожиженного слоя, эффективную излучательную способность неизотермического слоя и распределение температуры вблизи поверхности теплообменника при заданных свойствах частиц, теплообменника, расширении слоя.